OTPORNOST MATERIJALA

VEŽBE BR. 3

AKSIJALNO NAPREZANJE

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DEFINICIJA

- AKSIJALNO NAPREZANJE NASTAJE AKO SILA DELUJE U PRAVCU ŠTAPA, UPRAVNO NA POPREČNI PRESEK, KOJI SA OSOM ŠTAPA GRADI PRAV UGAO. OVO NAPREZANJE MOŽE SE JAVITI KAO ZATEZANJE ILI PRITISAK.

FORMULE

DILATACIJA

$$\varepsilon = \frac{\Delta l}{l}$$

HUKOV ZAKON

$$\sigma = \varepsilon * E$$

HUKOV ZAKON ZA IZDUŽENJE

$$\Delta l = \frac{Fl}{AE}$$

NORMALNI NAPON

$$\sigma = \frac{F}{A}$$

POPREČNA DILATACIJA

$$\varepsilon_p = -\mu\varepsilon$$

FORMULE

- Dimenzionisanje se vrši prema normalnom naponu, koji mora biti manji od dozvoljenog napona σ_{doz}.

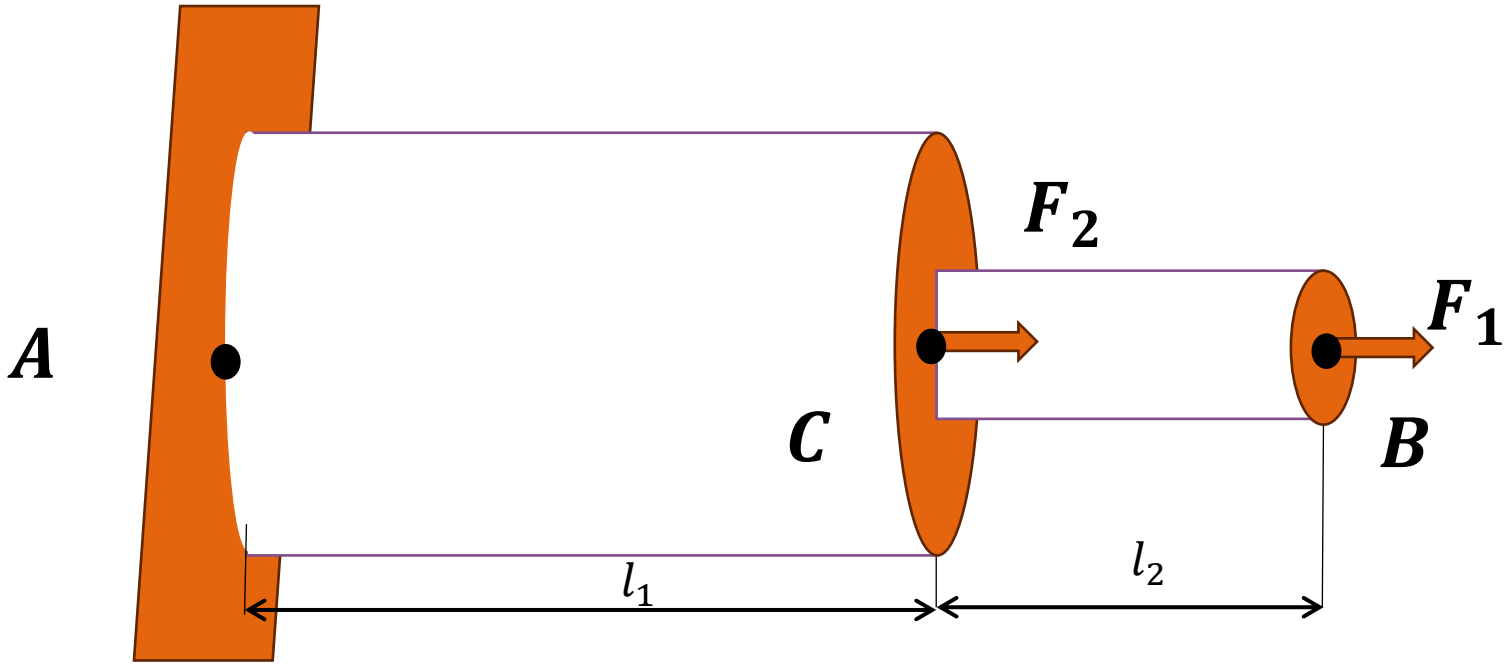
$$\sigma = \frac{F}{A} \leq \sigma_{doz}$$

$$A \geq \frac{F}{\sigma_{doz}}$$

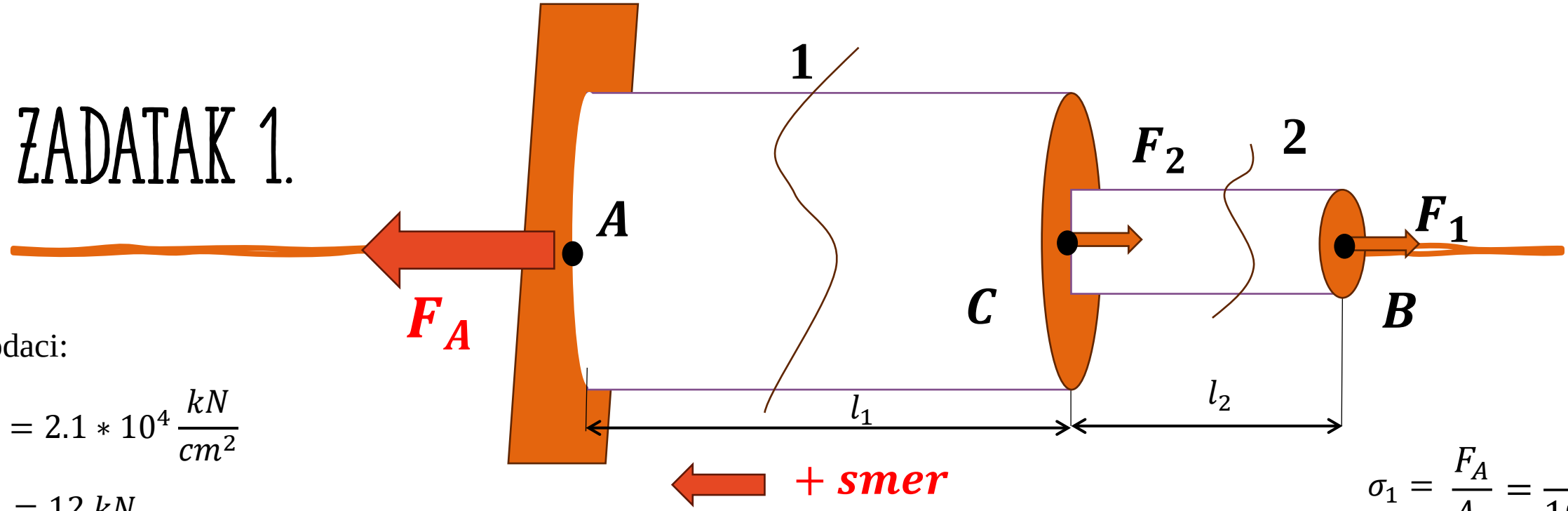
ZADACI

ZADATAK 1.

- Štap AB kvadratnog poprečnog preseka izrađen je od materijala čiji je modul elastičnosti $2 * 10^4 \frac{kN}{cm^2}$ i opterećn je kao na slici. Izračunati napone u oba dela štapa i njegovo ukupno izduženje ako je: $F_1 = 12\ kN, F_2 = 20\ kN, a_1 = 32\ mm, a_2 = 16\ mm, l_1 = 320\ mm\ i\ l_2 = 640\ mm$.



ZADATAK 1.



Podaci:

$E = 2.1 * 10^4 \frac{kN}{cm^2}$

$F_1 = 12\ kN$

$F_2 = 20\ kN$

$a_1 = 32\ mm$

$a_2 = 16\ mm$

$l_1 = 320\ mm$

$l_2 = 640\ mm$

$\Delta l = ?\quad \sigma_1 = ?\quad \sigma_2 = ?$

$$\sum_{i=1}^{n} X_i = 0$$

$$F_1 + F_2 - F_A = 0$$

$$F_A = F_1 + F_2$$

$$F_A = 32kN$$

$$\sigma = \frac{F}{A} \leq \sigma_{doz}$$

$$\sigma_1 = \frac{F_A}{A_1}$$

$$A_1 = a_1{}^2 = 10.24\ cm^2$$

$$\sigma_2 = \frac{F_A - F_2}{A_2}$$

$$A_2 = a_2{}^2 = 2.56\ cm^2$$

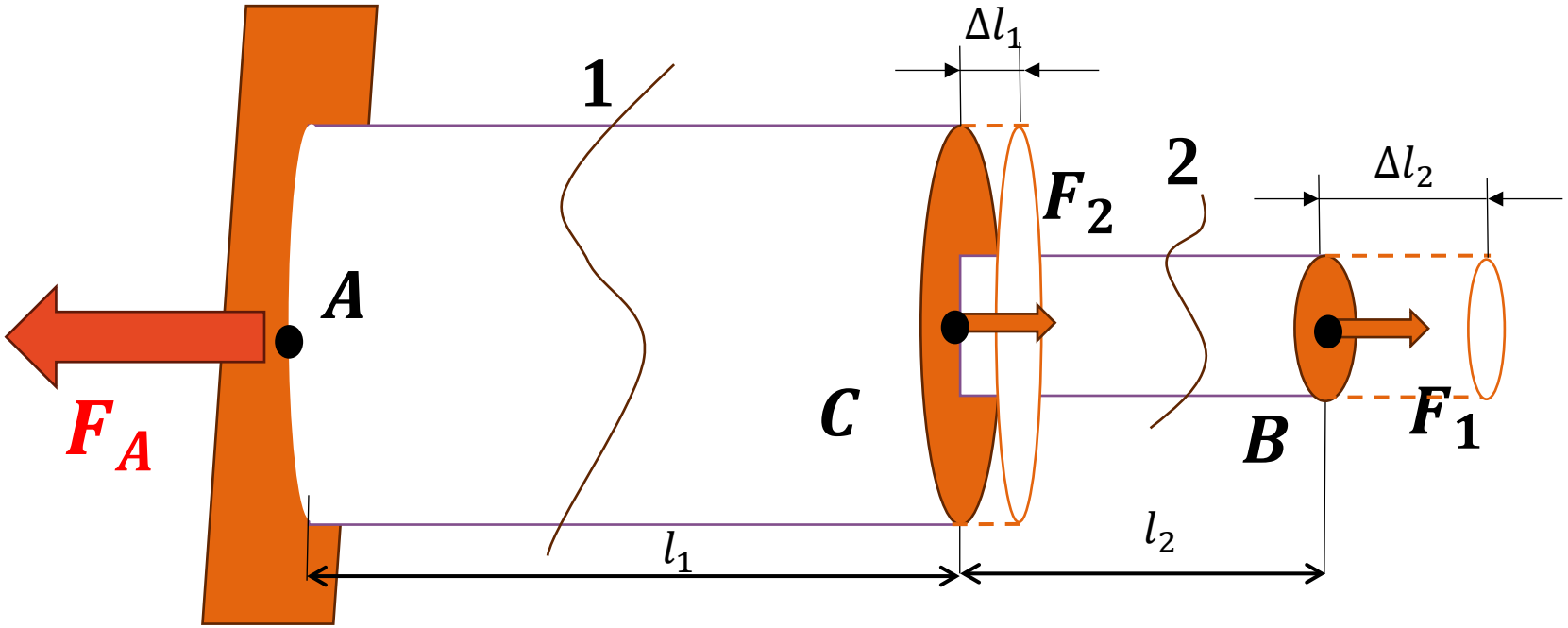
$$\sigma_1 = \frac{F_A}{A_1} = \frac{32}{10.24}$$

$$\sigma_1 = 3.125 \frac{kN}{cm^2}$$

$$\sigma_2 = \frac{F_A - F_2}{A_2} = \frac{32 - 20}{2.56}$$

$$\sigma_2 = 4.6875 \frac{kN}{cm^2}$$

ZADATAK 1.



$$\Delta l = \Delta l_1 + \Delta l_2$$

$$\Delta l = \frac{Fl}{AE}$$

$$\Delta l_1 = \frac{F_A l_1}{A_1 E} = \frac{32 * 3.2}{10.24 * 2 * 10^4} = 0.0005\ cm$$

$$\Delta l_2 = \frac{(F_A - F_2) l_2}{A_2 E} = \frac{12 * 6.4}{2.56 * 2 * 10^4} = 0.0015\ cm$$

ZADATAK 2.

Štap AB promenljivog poprečnog preseka, uklešten na oba kraja, kao što je prikazano na slici, opterećen je aksijalnom silom F = 10 kN u preseku C. Odrediti:

a) otpore ukleštenja,

b) normalne napone u poprečnom preseku svakog raspona i

c) pomeranje tačke C

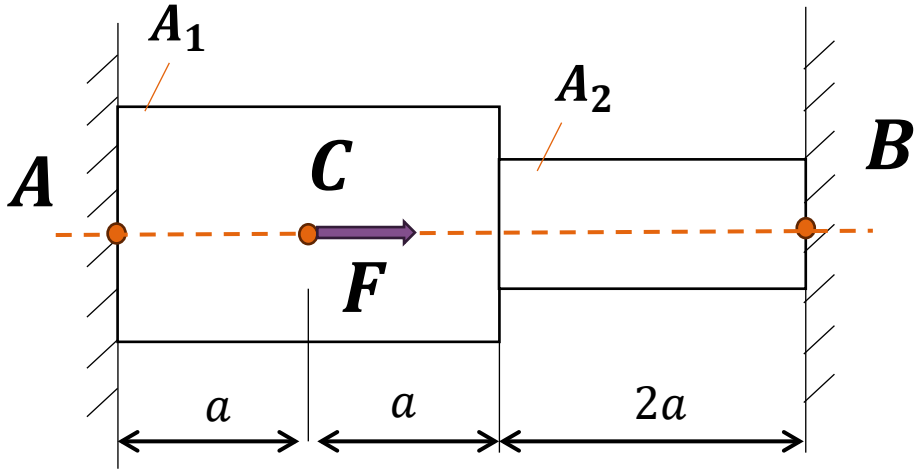
Podaci:

$E = 2.1 * 10^4 \frac{kN}{cm^2}$

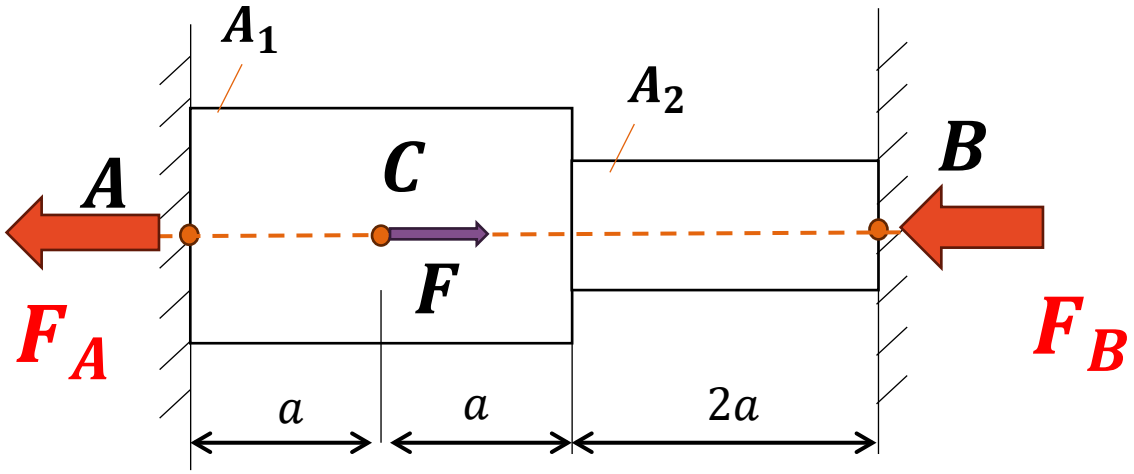
$A_1 = 6\ cm^2$

$A_2 = 4\ cm^2$

$a = 60\ cm$



ZADATAK 2.



$$\sum_{i=1}^{n} F_i = 0 \qquad \Delta l = \frac{Fl}{AE}$$

$$\mathrm{F} - F_A - F_B = 0$$

$$\mathrm{F} = F_A + F_B$$

$$F_A = F - F_B \quad F_B = F - F_A$$

$$\Delta l_{AB} = \Delta l_{AC} + \Delta l_{CD} + \Delta l_{DB} = 0$$

$$\frac{(F - F_B)a}{A_1 E} - \frac{(F_B)a}{A_1 E} - \frac{(F_B)2a}{A_2 E} = 0$$

$$\frac{Fa}{A_1 E} - \frac{2(F_B)a}{A_1 E} - \frac{(F_B)2a}{A_2 E} = 0$$

$$2\left(\frac{F_B}{A_2} - \frac{F_B}{A_1}\right) = \frac{F}{A_1}$$

$$2F_B\left(\frac{A_2 + A_1}{A_1 A_2}\right) = \frac{F}{A_1}$$

$$F_B = \frac{F}{2}\left(\frac{A_2}{A_2 + A_1}\right)$$

$$F_B = 2\ kN$$

$$F_A = 8\ kN$$

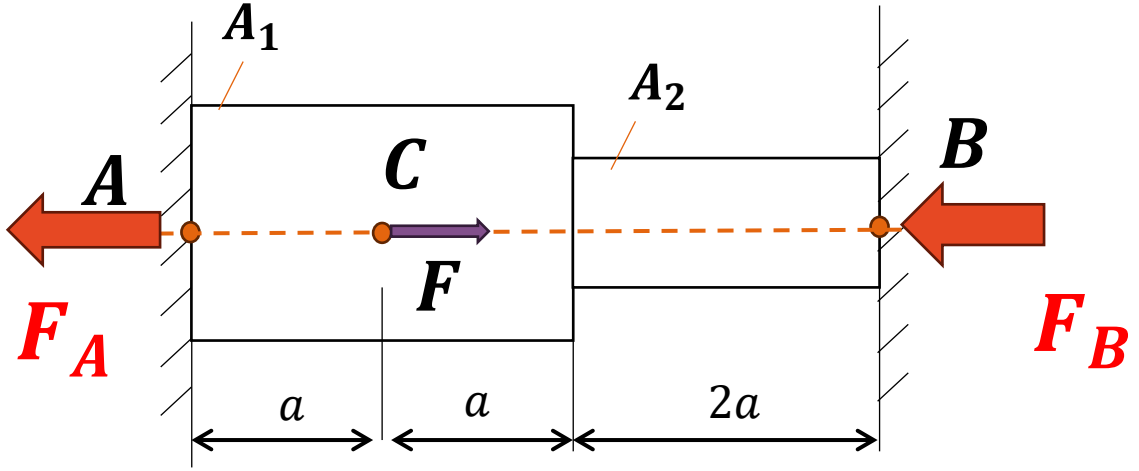
ZADATAK 2.

$b)$

$$\sigma_{AC} = \frac{F - F_B}{A_1} = \frac{10-2}{6} = 1.33\ \frac{kN}{cm^2}$$

$$\sigma_{CD} = \frac{-F_B}{A_1} = \frac{-2}{6} = -0.33\ \frac{kN}{cm^2}$$

$$\sigma_{DB} = \frac{-F_B}{A_2} = \frac{-2}{4} = -0.5\ \frac{kN}{cm^2}$$

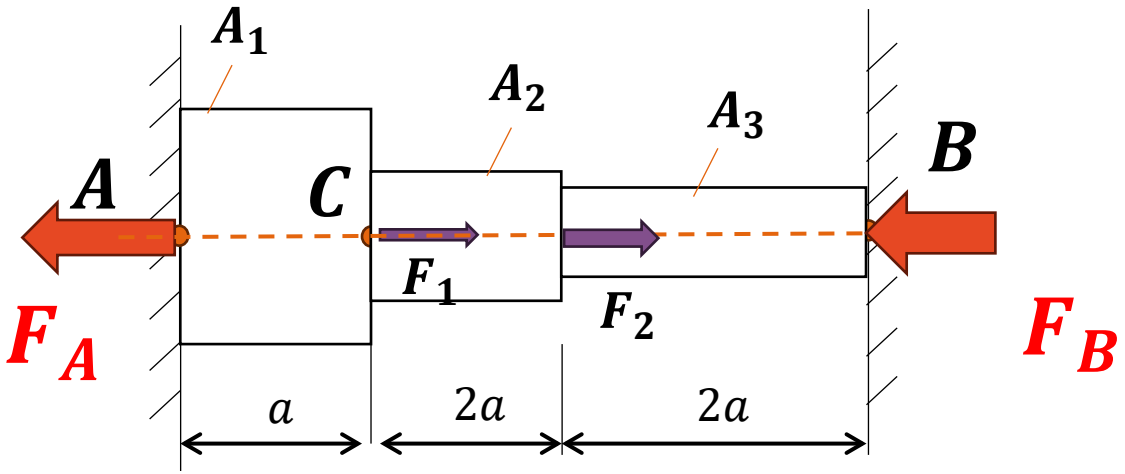


$c)$

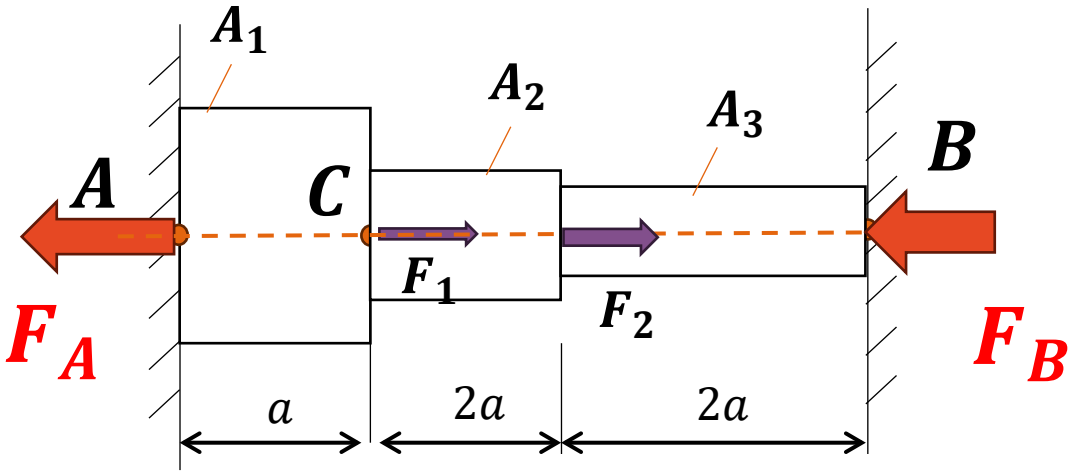
$$\Delta l_{AC} = \frac{(F - F_B)a}{A_1 E} = \frac{(10-2)60}{6 * 2.1 * 10^4} = 40 * 10^{-4}\ cm$$

ZADATAK 3.

Štap AB sastoji se iz tri dela i opterećen je kao na slici. Odrediti napone u sva tri dela štapa, ako je $E = 2 * 10^4 \frac{kN}{cm^2}, F_1 = 20\ kN, F_2 = 15\ kN, A_1 = \frac{3}{2}, A_2 = 3\ cm^2, A_3 = 6\ cm^2, a = 20\ cm.$



ZADATAK 3.



$$\sum_{i=1}^{n} F_i = 0 \qquad \Delta l = \frac{Fl}{AE}$$

$$F_1 + F_2 - F_A - F_B = 0$$

$$F_A = 35 - F_B$$

$$\Delta l_{AB} = \Delta l_{AC} + \Delta l_{CD} + \Delta\, l_{DB} = 0$$

$$\frac{(F_A)a}{A_1 E} + \frac{(F_A - F_1)2a}{A_2 E} + \frac{(F_A - F_1 - F_2)2a}{A_3 E} = 0$$

$$\frac{F_A * 20}{3 * 10^4} + \frac{40\,F_A - 800}{6 * 10^4} + \frac{20\,F_A - 700}{12 * 10^4} = 0$$

$$\frac{F_A * 10}{6 * 10^4} = \frac{1500}{6 * 10^4}$$

$$F_A = 15\ kN \qquad F_B = 20\ kN$$

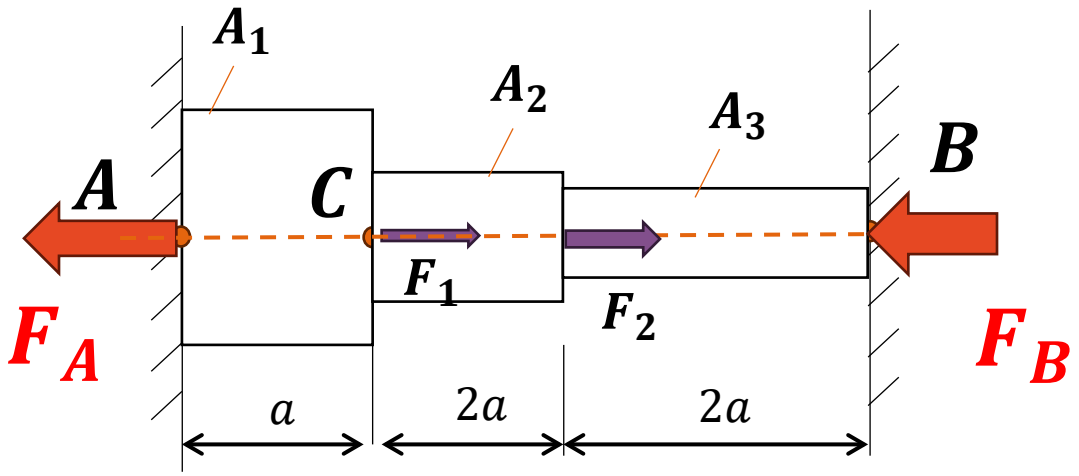
ZADATAK 3.

$b)$

$$\sigma_{AC} = \frac{F_A}{A_1} = \frac{15}{3/2} = 10\,\frac{kN}{cm^2}$$

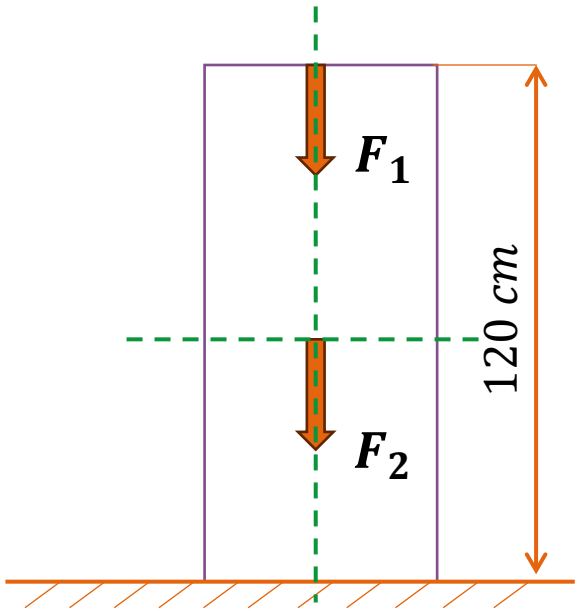
$$\sigma_{CD} = \frac{F_A - F_1}{A_2} = \frac{-5}{3} = -1.667\,\frac{kN}{cm^2}$$

$$\sigma_{DB} = \frac{F_A - F_1 - F_2}{A_3} = \frac{-20}{6} = -3.334\,\frac{kN}{cm^2}$$



ZADATAK 4.

- Stub kvadratnog poprečnog preseka,$a = 4\ cm$, i dužine 120 cm opterećen je aksijalnim silama $F_1 = 20\ kN\ i\ F_2 = 15\ kN$, na udaljenosti 0 i 60 cm od slobodnog kraja, izračunati napone u označenim presecima. Sopstvenu težinu stuba zanemariti.



Rešenje:

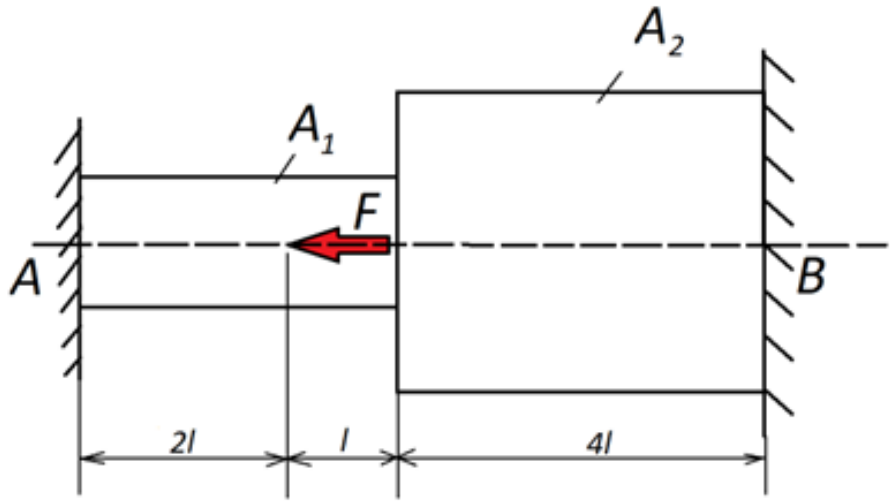
$$A = a^2 = 16\ cm^2$$

$$\sigma_1 = -1.25\ \frac{kN}{cm^2}$$

$$\sigma_2 = -2.1875\ \frac{kN}{cm^2}$$

ZADATAK 5.

Štap AB promenljivog poprečnog preseka, uklešten na oba kraja, kao što je prikazano na slici, opterećen je aksijalnom silom F=80 kN u preseku F. Odrediti otpore ukleštenja i normalne napone u poprečnom preseku svakog raspona. $E = 2 * 10^4 \frac{kN}{cm^2}, A_1 = 3cm^2, A_2 = 5\ cm^2, l = 2\ cm.$



HVALA NA PAŽNJI!